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A GENERAL ANALYTICAL SOLUTION TO THE EQUATION OF TRANSIENT FORCED CONVECTION WITH FULLY DEVELOPED FLOW

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Abstract—A general solution to the energy equation under zero wall temperature or zero heat flux boundary condition for the decay of an inlet and initial temperature distribution of an incompressible transient turbulent flow heat transfer between two parallel plates is given. It is shown that these solutions may then be used to obtain solutions due to unit steps in wall temperature or wall heat flux which is sufficient to sort out prescribed wall temperature and prescribed wall heat flux boundary condition. The results are confirmed experimentally by the frequency method. An experimental apparatus has been designed, built and used for this purpose.

NOMENCLATURE

$D(=a+\varepsilon_h)$, effective diffusivity;
$D_e(=2d)$, equivalent diameter;
$K(=k+\rho c_p \varepsilon_h)$, effective thermal conductivity of fluid;
T, temperature;
U, V, velocity components in x, y;
c_p, specific heat at constant pressure;
d, distance between parallel plates;
k, molecular thermal conductivity of fluid;
t, time;
$\bar{u}$, average velocity;
x, y, cartesian coordinates (x-flow direction, y-distance from wall);
a, $= k/\rho c_p$, thermal diffusivity;
Pr, $= c_p \mu/k$, Prandtl number;
Re, $= 2\bar{u}d/\nu$, Reynolds number;
β, inlet frequency;
α, γ, parameters;
δ, phase lag;
ε_h, eddy diffusivity for heat.

1. INTRODUCTION

THE STUDY of unsteady forced convection heat transfer in tubes and ducts has recently become of greater importance in connection with the control of modern high performance heat transfer devices. Literature on thermal transient is limited but increasing. Some of the important contributions are listed in the references [1–21].

Previous solutions to the problem of transient forced convection have assumed constant velocity across the flow, in association with either constant conductivity or constant temperature across the flow. Alternatively a form of integral approximation across the flow has been used.

For steady-state forced convection studies the concept of an "effective thermal conductivity" ranging in a known manner across the flow has given good agreement with experiment. The concept is here extended to the transient case, and a comparison made with experimental results at various Reynolds numbers.

The purpose of the work is firstly to make a further and more severe check on the validity of effective thermal conductivity, and secondly to produce a more accurate method of evaluating transients in thermal processes.

It is convenient to consider first the case of either zero temperature along the walls of the tube or duct or zero heat flux along the walls. These solutions may then be manipulated to obtain solutions due to unit steps in wall temperature or wall heat flux and by superposition of step solutions any arbitrary boundary condition may be studied.

Besides the boundary conditions it is also necessary to satisfy initial conditions and entry conditions. Consideration of these is best deferred until the general form of solution is seen. In this paper the algebra will be written for the case of duct geometry; the case of tube geometry will be apparent.

2. SOLUTIONS FOR ZERO BOUNDARY CONDITION

Consider a parallel plate channel whose sides are separated by a distance d. The parallel plate channel under consideration is shown in Fig. 1. Axial distances from the entrance section are measured by coordinate x, while transverse distances are measured by y.

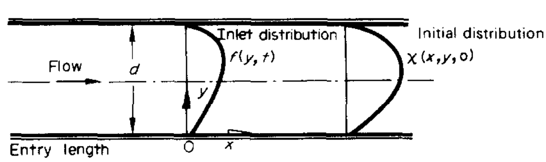

FIG. 1. Geometry of the system.

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Starting point of the analysis is the unsteady equation for a fully-developed hydrodynamic flow in a parallel-sided duct.

and
$$\rho c_p\left(\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial x}\right)=\frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right) \tag{1}$$

where K is the effective thermal conductivity.

The system satisfying the equation (1) is subjected to the following restrictions:

(a) The fluid velocity profile does not vary along the length of the duct.
(b) The mean velocity in the y-direction is zero.
(c) The effective thermal conductivity K consisting of the superimposed effects of molecular conductivity, will be assumed to depend on the distance y only.
(d) The effective conductivity K will be assumed independent of temperature T. This assumption is valid for a K which is largely governed by turbulence, it is not particularly good if temperature variations are large, due to consequent changes in ρ and c_p.
(e) Frictional dissipation of energy is negligible.
(f) Axial diffusion is negligible with respect to bulk transport in the x-direction. This is a reasonable assumption when Péclet number exceeds 100 [4].

In order to keep the problem linear, it is necessary to assume that the product $c_p\rho$ is independent of temperature T; in steady-state studies this has not been found very limiting, and it will be no more limiting in the transient state.

The equation may then be written

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial x}=\frac{\partial}{\partial y}\left(D\frac{\partial T}{\partial y}\right) \tag{1a}$$

where D is the effective thermal diffusivity, dependent on y. It may easily be confirmed that the following expressions satisfy equation (1a).

$$\begin{aligned}T(x,y,t)&=[P\sin(\beta t-\delta x)+Q\cos(\beta t-\delta x)]\,e^{-\alpha x}\\ T(x,y,t)&=[P\cos(\beta t-\delta x)-Q\sin(\beta t-\delta x)]\,e^{-\alpha x}\end{aligned} \tag{2}$$

where P and Q satisfy

$$\begin{aligned}\frac{d}{dy}\left(D\frac{dP}{dy}\right)&=-\alpha UP+(\delta U-\beta)Q\\ \frac{d}{dy}\left(D\frac{dQ}{dy}\right)&=-\alpha UQ-(\delta U-\beta)P\end{aligned} \tag{2a, b}$$

and

$$\begin{aligned}T(x,y,t)&=[R\sin(\delta x-\beta t)+S\cos(\delta x-\beta t)]\,e^{-\gamma t}\\ T(x,y,t)&=[R\cos(\delta x-\beta t)-S\sin(\delta x-\beta t)]\,e^{-\gamma t}\end{aligned} \tag{3}$$

where R and S satisfy

$$\begin{aligned}\frac{d}{dy}\left(D\frac{dR}{dy}\right)&=-\gamma R-(\delta U-\beta)S\\ \frac{d}{dy}\left(D\frac{dS}{dy}\right)&=-\gamma S+(\delta U-\beta)R.\end{aligned} \tag{3a, b}$$

Solutions of equations (2a, b) give rise to four pairs of functions $P(y)$ and $Q(y)$. Consider the case of a duct with walls at $y=0$ and $y=d$ and a boundary condition of zero temperature at each wall. It is then necessary that $P(y)$ and $Q(y)$ should vanish at each wall. From the four pairs of functions three pairs still satisfying equation (2a, b) can be formed by linear combination such that $P(y)$ vanishes at $y=0$. From these three pairs two pairs can be formed such that $Q(y)$ also vanishes at $y=0$.

It is seen that from expression (2) that there is a relationship between these two pairs of functions. If the first pair is denoted by $P(y)$, $Q(y)$ then the second pair is denoted by $-Q(y)$, $P(y)$; in these pairs, the coefficient of the sine term is written first, and is followed by the coefficient of the cosine term. By linear combination, a single pair may be formed such that the coefficient of the sine term vanishes at $y=d$.

The pair obtained is

$$Q(d)P(y)-P(d)Q(y);\ Q(d)Q(y)+P(d)P(y).$$

At $y=d$ the first function in this pair vanishes. The second function in the pair becomes $Q^2(d)+P^2(d)$ at $y=d$, and can only vanish there, if $P(d)=Q(d)=0$. Two conditions thus arise, and it is useful to consider these as specifying α and δ in terms of β. In fact for a given flow regime, that is for a given Reynolds number, every value of β gives rise to a set of eigenvalues for α and δ and a corresponding set of eigenfunctions for P and Q are also obtained.

Precisely identical reasoning leads to the same conclusion (with different numerical values for α and δ) if either or both boundary conditions of zero temperature are replaced by boundary conditions of zero heat flux. The same reasoning may also be applied to functions $R(y)$ and $S(y)$ with similar boundary conditions. In this case it is useful to regard β and γ as specified in terms of δ; eigenvalues and eigenfunctions again arise.

If subscripts m and n represent different eigenvalues and the corresponding eigenfunctions, it may be shown that

$$\int_0^d U(P_mP_n-Q_mQ_n)\,dy=0$$
$$\int_0^d U(P_mQ_n+P_nQ_m)\,dy=0$$
and
$$\int_0^d (R_mR_n-S_nS_m)\,dy=0$$
$$\int_0^d (R_mS_n+R_nS_m)\,dy=0.$$

In view of this orthogonality it is reasonable to assume that the set of functions P_n, Q_n is complete, and that the set R_n, S_n is also complete.

3. ENTRY CONDITIONS AND INITIAL CONDITIONS

If the functions $P(y)$ and $Q(y)$ are taken as normalized in the same way, the most general solutions based on expression (2) and (3) which may be written down, are

$$\sum_n \int_0^\infty A_n(\beta)[P_n(\beta, y)\sin(\beta t-\delta_n x) + Q_n(\beta, y)\cos(\beta t-\delta_n x)]\,e^{-\alpha_n x}\,d\beta$$
$$+\sum_n \int_0^\infty B_n(\beta)[P_n(\beta, y)\cos(\beta t-\delta_n x) - Q_n(\beta, y)\sin(\beta t-\delta_n x)]\,e^{-\alpha_n x}\,d\beta \quad (4a)$$

$$\sum_n \int_0^\infty A_n'(\delta)[R_n(\delta, y)\sin(\delta x-\beta t) + S_n(\delta, y)\cos(\delta x-\beta t)]\,e^{-\gamma t}\,d\delta$$
$$+\sum_n \int_0^\infty B_n'(\delta)[R_n(\delta, y)\cos(\delta x-\beta t) - S_n(\delta, y)\sin(\delta x-\beta t)]\,e^{-\gamma t}\,d\delta \quad (4b)$$

putting $x = 0$ in expression (4a), and $t = 0$ in expression (4b)

$$\sum_n \int_0^\infty A_n(\beta)[P_n(\beta, y)\sin\beta t+Q_n(\beta, y)\cos\beta t]\,d\beta$$
$$+\sum_n \int_0^\infty B_n(\beta)[P_n(\beta, y)\cos\beta t-Q_n(\beta, y)\sin\beta t]\,d\beta \quad (5a)$$

and

$$\sum_n \int_0^\infty A_n'(\delta)[R_n(\delta, y)\sin\delta x+S_n(\delta, y)\cos\delta x]\,d\delta$$
$$+\sum_n \int_0^\infty B_n'(\delta)[R_n(\delta, y)\cos\delta x-S_n(\delta, y)\sin\delta x]\,d\delta. \quad (5b)$$

Recalling the assumption about the completeness of the set of functions P_n, Q_n and remembering Fourier's integral theorem, inspection of expression (5a) suggests that it might be capable of representing a perfectly general entry condition from $-\infty$ to $+\infty$ in time, and from 0 to d in y. If this is so, expression (4a) then represents the temperature distribution in the duct due to this entry condition.

Let $f(y, t)$ be a general inlet condition with $0 < y < d$; $-\infty < t < \infty$. Then by Fourier integral theorem

$$f(y, t) = \int_0^\infty C(\beta, y)\sin\beta t\,d\beta + \int_0^\infty D(\beta, y)\cos\beta t\,d\beta$$

where

$$C(\beta, y) = \frac{1}{\pi}\int_{-\infty}^{+\infty} f(y, u)\sin\beta u\,du$$

and

$$D(\beta, y) = \frac{1}{\pi}\int_{-\infty}^{+\infty} f(y, u)\cos\beta u\,du.$$

Comparing the above with expression (5a), it remains to find A_n and B_n satisfying

$$\sum_n [A_n(\beta)P_n(\beta, y)-B_n(\beta)Q_n(\beta, y)] = C(\beta, y)$$

$$\sum_n [A_n(\beta)Q_n(\beta, y)+B_n(\beta)P_n(\beta, y)] = D(\beta, y).$$

Suppressing the β, it may be shown that

$$A_n\int_0^d U[P_n^2(y)-Q_n^2(y)]\,dy-2B_n\int_0^d UP_n(y)Q_n(y)\,dy = \int_0^d UC(y)P_n(y)\,dy-\int_0^d UD(y)Q_n(y)\,dy$$
$$2A_n\int_0^d UP_n(y)Q_n(y)\,dy+B_n\int_0^d U[P_n^2(y)-Q_n^2(y)]\,dy = \int_0^d UC(y)Q_n(y)\,dy+\int_0^d UD(y)P_n(y)\,dy. \quad (6)$$

From which A_n and B_n may be found. A method of solution for a duct extending from zero to infinity in x with an entry condition known from minus infinity to plus infinity in time has therefore been established. This solution is based on expression (2). As a special case of the decay of the general inlet temperature distribution, in [20] a solution for the decay of inlet temperature distribution which varies sinusoidally in time is presented; and in [18], a specified consideration is given to laminar flow in a parallel plate channel for time varying inlet temperature and participating walls.

Expressions (3) similarly form the basis of a method of solution for a duct extending from zero to infinity in time with an initial condition known from minus infinity to plus infinity in x. Let $\chi(x, y)$ be a general initial condition with $0 < y < d$, $-\infty < x < \infty$, then by Fourier integral theorem

$$\chi(x, y) = \int_0^\infty C_n'(\delta, y)\sin\delta x\,d\delta + \int_0^\infty D_n'(\delta, y)\cos\delta x\,d\delta$$

where $C_n'(x, y)$ and $D'(\delta, y)$ can be determined depending on the nature of the orthogonalities involving R and S.

The case most often occurring in practice is a duct which may be deemed to extend from zero to infinity in x and also from zero to infinity in time. The solution to this problem is found by adding two solutions. These are,

(a) The case of duct extending from zero to infinity in x, with zero entry temperature for all $t < 0$, and the correct entry temperature variation for $t > 0$.

(b) The case of a duct extending from minus infinity to plus infinity in x, with zero initial temperature for all $x < 0$, and the correct initial temperature distribution for $x > 0$.

In (a) the zero entry temperature for all $t < 0$ means that at $t = 0$ the temperature given by (a) is zero in the duct. In (b) the zero initial temperature for all $x < 0$ means that at $x = 0$ the temperature given by (b) is zero for all $t > 0$.

4. SOLUTIONS FOR ARBITRARY BOUNDARY CONDITIONS

Boundary conditions of prescribed wall temperature and prescribed wall heat flux may be built up by superposition from solutions for unit steps in wall temperature or wall heat flux respectively.

The complete solution is in general then a sum of three terms. These are the initial condition, the entry condition, and the boundary condition. If the initial condition is known from minus infinity to plus infinity in x, then no entry condition arises. If the entry condition is known from minus infinity to plus infinity in time then no initial condition arises.

In this section the case of a unit step in wall temperature of a semi-infinite duct will be found. The step is assumed to occur at zero time and to extend over all $x > 0$. From this any arbitrary wall temperature can be found. A similar process applies to wall heat flux.

Let ϕ be a solution for unit entry condition for $t > 0$, with zero initial condition for $x > 0$, and zero boundary condition. Then

$$\begin{array}{llll} \phi = 1 & \text{at } x = 0 & \text{for all} & t > 0 \\ \phi = 0 & \text{at } t = 0 & \text{for all} & t > 0 \\ (\text{i.e. } \phi = 0 & \text{at } x = 0 & \text{for all} & t < 0) \\ \phi = 0 & \text{at } y = 0 & \text{and} & Y = d \end{array}$$

let Ψ be a solution for unit initial for $x > 0$, with zero entry condition for $t > 0$ and zero boundary conditions. Then

$$\begin{array}{llll} \psi = 0 & \text{at } x = 0 & \text{for all} & t > 0 \\ (\text{i.e. } \psi = 0 & \text{at } t = 0 & \text{for all} & x < 0) \\ \psi = 1 & \text{at } t = 0 & \text{for all} & x > 0 \\ \psi = 0 & \text{at } Y = 0 & \text{and} & y = d \end{array}$$

Consider

$$\begin{array}{llll} \theta = 1 - \phi - \Psi & & & \\ \theta = 0 & \text{at } x = 0 & \text{for all} & t > 0 \\ \theta = 0 & \text{at } t = 0 & \text{for all} & x > 0 \\ \theta = 1 & \text{at } y = 0 & \text{and} & y = d. \end{array}$$

Therefore θ is the required solution for a step in wall temperature with zero initial condition and zero entry condition. Similar techniques apply to more complicated cases.

5. COMPARISON WITH EXPERIMENT

The form of equations (2) and (3) suggest that the results can be best confirmed experimentally by the frequency method. By this method, the parameters and eigenvalues appearing in the general solution can be determined. An apparatus has been designed and used for the frequency analysis of an inlet temperature distribution for forced convection with fully developed flow between two parallel plates. This apparatus and technique was partly described in [20]; but the full description of the apparatus and frequency analysis methods will be given in another paper. The boundary conditions used were zero temperature at one side of the duct and zero heat flux at the other. The response to the sinusoidal variation of heat input has been recorded on a strip-chart. From these recordings the amplitudes at various points along the duct have been presented in graphical forms for various inlet frequencies; phase lags in the response to the sinusoidal variation of heat input have been determined along the duct for various values of Reynolds numbers and inlet frequencies [20]. Further experiments have been conducted for the decay of inlet temperature distribution along the duct.

As the frequency β of the sine wave at the inlet was varied, it was possible from these measurements to obtain the values of α and δ for the lowest eigenfunction after the higher eigenfunctions excited had damped out, Figs. 2 and 3.

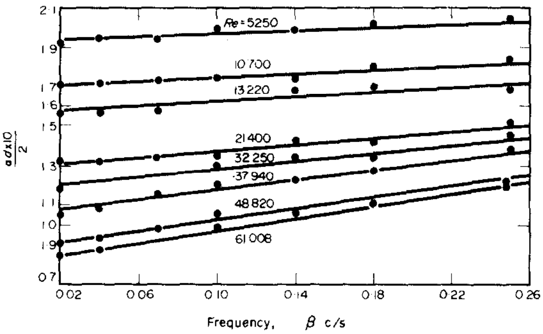

FIG. 2.

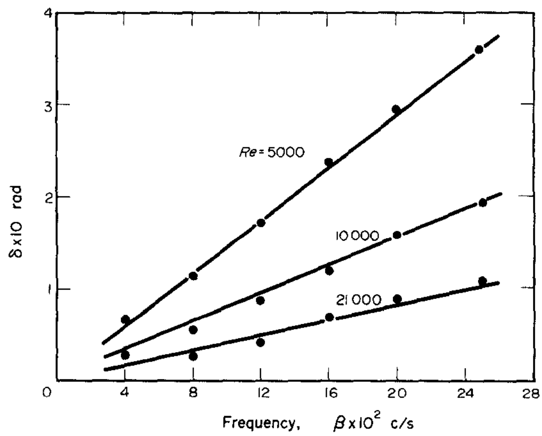

FIG. 3.

6. CONCLUDING REMARKS

Solutions determine the temperature distribution as a function of time and space in the form of the sum of two infinite series, each term of which consists of a product of an exponential and two functions. These solutions mean that any initial temperature or inlet temperature distribution may be regarded as the result of the superposition of a number of modes of periodic distributions. Each decays exponentially with time or with the distance along the duct.

From the experimental results, it is seen that the decay of the inlet temperature distribution near the entrance region is not a single exponential. It consists of modes of higher frequency. The basic mode of the inlet temperature varies exponentially along the channel and the value of the temperature at a given point depends on the inlet frequency and Reynolds number. For given fluid as Reynolds number increases decay decreases.

Phase lags have been found to be increasing linearly with distance. Phase lag increases as inlet frequency increases and decreases as Reynolds number increases.

The apparatus available did not permit the determination of γ, but construction of a new apparatus is being under consideration.

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UNE SOLUTION ANALYTIQUE GENERALE DE L'EQUATION DE LA CONVECTION FORCEE INSTATIONNAIRE EN ECOULEMENT ETABLI

Résumé—On donne la solution générale de l'équation d'énergie pour la décroissance d'une distribution initiale de température d'admission dans un écoulement turbulent incompressible instationnaire entre deux plaques parallèles avec des conditions aux limites à température nulle ou à flux thermique nul à la paroi. On montre que ces solutions peuvent alors être utilisées pour obtenir les solutions correspondant à des échelons unité de la température de paroi ou du flux pariétal, suffisantes pour choisir les conditions aux limites à température de paroi imposée ou à flux pariétal imposé. Les résultats sont confirmés expérimentalement à l'aide de la méthode de fréquence. Un appareil expérimental a été conçu, réalisé et utilisé dans ce but.

ALLGEMEINE LÖSUNG DER GLEICHUNG FÜR ERZWUNGENE KONVEKTION IM ÜBERGANGSBEREICH BEI VOLL AUSGEBILDETER STRÖMUNG

Zusammenfassung—Es wird eine allgemeine Lösung der Energiegleichung für den Abbau einer anfänglichen Eintrittstemperaturverteilung einer inkompressiblen Strömung im Übergangsbereich bei Wärmeaustausch zwischen zwei parallelen Platten angegeben.

Es wird gezeigt, daß diese Lösungen dann benutzt werden können, Lösungen für schrittweise geänderte Wandtemperaturen oder Wärmeströme an der Wand zu erhalten, so daß es möglich wird, nach vorgegebenen Randbedingungen für Temperatur und Wärmestrom an der Wand auszulesen. Die Ergebnisse werden experimentell mit Hilfe des Frequenzen-Verfahrens bestätigt. Zu diesem Zweck wurde ein Versuchsapparat entworfen und aufgebaut.

ОБЩЕЕ АНАЛИТИЧЕСКОЕ РЕШЕНИЕ УРАВНЕНИЯ НЕСТАЦИОНАРНОЙ ВЫНУЖДЕННОЙ КОНВЕКЦИИ ПРИ ПОЛНОСТЬЮ РАЗВИТОМ ТЕЧЕНИИ

Аннотация — Представлено общее решение уравнения энергии при граничных условиях нулевой температуры стенки или нулевого теплового потока для затухающего распределения температуры на входе и начальной температуры нестационарного турбулентного потока несжимаемой жидкости между двумя параллельными пластинами. Показано, что эти решения, благодаря единичным скачкам в температуре стенки или теплового потока на стенке, могут быть затем использованы для получения решений с граничными условиями заданной температуры стенки и заданного теплового потока на стенке. Результаты подтверждаются экспериментально с помощью частотного метода, для чего была спроектирована и построена экспериментальная установка.